Gravitational Wave Sources May Be "Further" Than We Think

ASGHAR QADIR[†]

[†]Centre for Advanced Mathematics and Physics (CAMP)
National University of Sciences and Technology
Campus of the College of Electrical and Mechanical Engineering
Peshawar Road, Rawalpindi, Pakistan

aqadirmath@yahoo.com

Abstract. It has been argued that the energy content in time varying spacetimes can be obtained by using the approximate Lie symmetries of the geodesics equations in that spacetime. When applied to cylindrical gravitational waves, it gives a self-damping of the waves. According to this proposal the energy of the waves go to zero asymptotically as the radial distance to the two-thirds power. If true, this would mean that the estimates for the sensitivity of the detectors for the various sources would have to be revised.

Key words: Gravitational waves; SN1987A; Energy; Approximate Lie symmetries

In the early days of Relativity there were doubts raised about the reality of gravitational waves as they were solutions of vacuum field equations. Weber and Wheeler demonstrated that they would impart momentum to test particles in their path [1]. This demonstration was extended to test particles in the path of plane gravitational waves by Ehlers and Kundt [2]. A general closed formula for the momentum imparted to test particles in an arbitrary spacetime was given by Qadir and Sharif [3].

Though it seemed obvious that "the waves must carry energy", there was no clear measure available for the energy carried by them. This is because there is generally no energy conservation in Relativity and hence no definition of mass or energy. One way to avoid this problem is by defining a stress-energy "pseudo-tensor" (see for example [4]), but it is observer dependent and not generally accepted for defining the energy by relativists (see for example the discussion of the pseudo-tensor in [5]). A deeper approach is to define some measure for the breaking of time translational symmetry [6, 7, 8, 9, 10, 11, 12] and use that to provide a measure for the energy density in the spacetime. None of these attempts showed unambiguous success as they did not lead to any basic new physical insights or predictions.

Unlike electromagnetic waves, gravitational waves undergo-self interaction due to the non-linearity of Relativity. This self-interaction could, in principle cause damping like Landau damping of electromagnetic waves [13], due to their interaction with matter, or cause an enhancement such as might be expected on the basis of the work on colliding gravitational waves [14, 15], where singularities are formed due to the interaction between two plane gravitational waves.

One approximate symmetry proposal is the use of broken (or approximate) Lie symmetries [16, 17] of the geodesic equations [18]. The breaking is taken to be by terms involving a small parameter whose powers, higher than some chosen value, can be neglected. It was found that though with the first order approximate symmetries no new insight is obtained [19], for the second order the timelike symmetry picks up a scaling factor [20, 21] independent of the strength of the breaking. It may, thus, provide a genuine measure for the energy content of gravitational waves. This proposal has been implemented for cylindrical gravitational waves [22]. It yields a prediction that the self-interaction of the waves *attenuates* them by a factor $\sim \sqrt{(c/\rho\omega)}$, where ρ is the radial distance from the source and ω is the frequency of the wave. Note that, $\rho\omega/c = n$ is the number of wavelengths in the distance from the source. The longest waves will, thus, contribute the most in detection.

The purpose of this note is to examine the observational implications of this attenuation of gravitational waves implicit in the approximate symmetry proposal and look for predictions that can be tested.

The obvious implication of the predicted behaviour of energy is that our expectation of how the energy in the wave decreases with the distance of the source and the frequency of the wave (and hence the length of the antenna) will be very seriously overestimated. One can view this reduction alternatively as an effective increase of the distance of the source due to the warping of the geometry of the spacetime by the wave. However, one can not simply take the reduction factor to be $1/\sqrt{n}$, as that is found for cylindrical waves. There are no infinite axially symmetric sources in Nature to emit genuine cylindrical waves. Nor is it absolutely clear that a spherical, or slightly aspherical, source will give the same attenuation factor.

An interesting source to consider is SN1987A, which had a 0.1 asphericity in its explosion [23]. One could regard the resulting waves as, in some sense, combined "spherical" and cylindrical waves. Crudely, then, one could take a cylindrical component of 0.1 of the total and assume that it has the predicted attenuation factor. The axis of the explosion is estimated to make an angle of $\pi/4 = 45^o$ with our line of sight. The energy of spherical waves decreases as the square of the distance in normal geometry. It seems reasonable to assume that in the warped geometry there will be an attenuation $\geq \sqrt{(c/\rho^3\omega)}$. This reduction is enormous for SN1987A, which is at a distance of ~ 51.4 *kpc*. For an interferometer with an arm of 5.3 *km* the reduction factor comes out to be about a million! For the Hulse-Taylor [24] binary pulsar it would be about $300,000$ for the interferometer. Note that due to the frequency dependence of the attenuation, the long base-line interferometers will be better to look for gravitational waves. For a bar detector the above attenuation factor would be up by a factor of about thirty. *There is a clear prediction that the waves from this pulsar will not be seen by the currently planned detectors.*

Weber claimed to have observed a signal on his *non*-cryogenic bar detector synchronous with the light and neutrino burst from SN1987A [25]. Since his (standard) theory of the

bar detector [26] leads to energy requirements incompatible with the energy output of SN1987A ($\sim 10^{53}$ $ergs$), this claim was generally regarded as untenable. Indeed, Weber tried to re-vamp his theory of the detector to provide much greater efficiency of detection, using a claimed quantum coherence effect [27, 28]. While Bassan did not believe the claim, he admitted to having seen the signal found by Weber, as he was visiting Weber's laboratory at the time [29]. If our proposal is borne out, then Weber's claim of a sensitivity a billion times greater than that provided by his original theory would get reduced to an effective thousand — still far from sufficient to provide observability.

Of course, there are no exact spherical gravitational wave solutions available. However, there are Nutku's spherical wave solutions "with strings attached" [30]. The approximate Lie symmetry analysis could be applied to them to obtain the attenuation factor in that case. This analysis is vital for a more precise test of the proposal.

A detailed analysis of the sensitivity requirements for various sources according to their strengths and distances will be presented subsequently [31].

Acknowledgments

I am grateful to F. De Paolis for help in obtaining information regarding SN1987A.

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